Chapter 9: Momentum and Its Conservation
BIG IDEA

If the net force on a closed system is zero, the total momentum of that system is conserved.
MAIN IDEA
An object’s momentum is equal to its mass multiplied by its velocity.

Essential Questions
• What is impulse?
• What is momentum?
• What is angular momentum?
Review Vocabulary

- **Angular velocity** the angular displacement of an object divided by the time needed to make the displacement

New Vocabulary

- Impulse
- Momentum
- Impulse-momentum theorem
- Angular momentum
- Angular impulse-angular momentum theorem
Impulse-Momentum Theorem

Click image to view the movie.
The right side of the equation $F \Delta t = m \Delta v$, $m \Delta v$, involves the change in velocity: $\Delta v = v_f - v_i$.

Therefore, $m \Delta v = m v_f - m v_i$.
The product of the object’s mass, $m$, and the object’s velocity, $v$, is defined as the **momentum** of the object. Momentum is measured in kg·m/s. An object’s momentum, also known as linear momentum, is represented by the following equation:

\[
\text{Momentum} \quad p = mv
\]

- The momentum of an object is equal to the mass of the object times the object’s velocity.
Recall the equation $F \Delta t = m \Delta v = m v_f - m v_i$. Because $m v_f = p_f$ and $m v_i = p_i$, you get:

$$F \Delta t = m \Delta v = p_f - p_i$$

The right side of this equation, $p_f - p_i$, describes the change in momentum of an object. Thus, the impulse on an object is equal to the change in its momentum, which is called the impulse-momentum theorem.
The impulse-momentum theorem is represented by the following equation.

**Impulse-Momentum Theorem**

\[ F \Delta t = p_f - p_i \]

- The impulse on an object is equal to the object’s final momentum minus the object’s initial momentum.
If the force on an object is constant, the impulse is the product of the force multiplied by the time interval over which it acts.

Because velocity is a vector, momentum also is a vector.

Similarly, impulse is a vector because force is a vector.

This means that signs will be important for motion in one dimension.
Let’s discuss the change in momentum of a baseball. The impulse that is the area under the curve is approximately 13.1 N·s. The direction of the impulse is in the direction of the force. Therefore, the change in momentum of the ball is also 13.1 N·s.
Because 1 N·s is equal to 1 kg·m/s, the momentum gained by the ball is 13.1 kg·m/s in the direction of the force acting on it.
• What is the momentum of the ball after the collision?
• Solve the impulse-momentum theorem for the final momentum. \( p_f = p_i + F\Delta t \)
The ball’s final momentum is the sum of the initial momentum and the impulse. Thus, the ball’s final momentum is calculated as follows.

\[ p_f = p_i + 13.1 \text{ kg}\cdot\text{m/s} \]

- Mass of the ball is 0.145kg and velocity, before collision, is -38m/s. Therefore, the baseball’s \( p_i = (0.145\text{kg})(-38\text{m/s}) = -5.5 \text{ kg}\cdot\text{m/s} \)

\[ = -5.5 \text{ kg}\cdot\text{m/s} + 13.1 \text{ kg}\cdot\text{m/s} = +7.6 \text{ kg}\cdot\text{m/s} \]
• What is the baseball’s final velocity? Because $p_f = mv_f$, solving for $v_f$ yields the following:

$$v_f = \frac{p_f}{m} = \frac{+7.6 \text{ kg}}{+0.145 \text{ kg}} = +52 \text{ m/s}$$
Impulse-Momentum Theorem (Cont.)

• What happens to the driver when a crash suddenly stops a car?

• An impulse is needed to bring the driver’s momentum to zero.

• A large change in momentum occurs only when there is a large impulse.
Impulse-Momentum Theorem (Cont.)

- A large impulse can result either from a large force acting over a short period of time or from a smaller force acting over a long period of time.
• According to the impulse-momentum equation, $F \Delta t = p_f - p_i$, the final momentum, $p_f$, is zero. The initial momentum, $p_i$, is the same with or without an air bag.

• Thus, the impulse, $F \Delta t$, also is the same.
Average Force

- A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21 s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes, or in 0.22 s if it hits a concrete wall. What is the impulse exerted on the vehicle? What average force is exerted on the vehicle in each of these stops?
Step 1: Analyze and Sketch the Problem

- Sketch the system.
- Include a coordinate axis and select the positive direction to be the direction of the velocity of the car.
- Draw a vector diagram for momentum and impulse.
Identify the known and unknown variables.

**Known:**

\[ m = 2200 \text{ kg} \quad \Delta t_{\text{gentle braking}} = 21 \text{ s} \]
\[ \mathbf{v}_i = +26 \text{ m/s} \quad \Delta t_{\text{hard braking}} = 3.8 \text{ s} \]
\[ \mathbf{v}_f = +0.0 \text{ m/s} \quad \Delta t_{\text{hitting a wall}} = 0.22 \text{ s} \]

**Unknown:**

\[ F_{\text{gentle braking}} = ? \]
\[ F_{\text{hard braking}} = ? \]
\[ F_{\text{hitting a wall}} = ? \]
Step 2: Solve for the Unknown
Determine the initial momentum, $p_i$, before the crash.

$$p_i = mv_i$$
Average Force (Cont.)

Substitute $m = 2200 \text{ kg}$, $v_i = +26 \text{ m/s}$

$$p_i = (2200 \text{ kg}) (+26 \text{ m/s})$$

$$= +5.7 \times 10^4 \text{ kg} \cdot \text{m/s}$$
Determine the final momentum, $p_f$, before the crash.

$$p_f = mv_f$$
Average Force (Cont.)

Substitute $m = 2200 \text{ kg}$, $v_f = +0.0 \text{ m/s}$

$$p_f = (2200 \text{ kg}) ( +0.0 \text{ m/s})$$

$$= +0.0 \text{ kg} \cdot \text{m/s}$$
Apply the impulse-momentum theorem to obtain the force needed to stop the vehicle.

\[ F \Delta t = p_f - p_i \]
Average Force (Cont.)

Substitute $p_f = 0.0 \text{ kg} \cdot \text{m/s}$, $p_i = 5.7 \times 10^4 \text{ kg} \cdot \text{m/s}$

$$F\Delta t = (+0.0\times10^4 \text{ kg} \cdot \text{m/s}) - (- 5.7\times10^4 \text{ kg} \cdot \text{m/s})$$

$$= -5.7\times10^4 \text{ kg} \cdot \text{m/s}$$

$$F = \frac{(- 5.7\times10^4 \text{ kg} \cdot \text{m/s})}{\Delta t}$$
Average Force (Cont.)

Substitute $\Delta t_{\text{gentle braking}} = 21$ s

$$F_{\text{gentle braking}} = \frac{(-5.7 \times 10^4 \text{ kg} \cdot \text{m/s})}{21 \text{ s}}$$

$$= -2.7 \times 10^3 \text{ N}$$
Average Force (Cont.)

Substitute $\Delta t_{\text{hard braking}} = 3.8 \text{ s}$

$$F_{\text{hard braking}} = \frac{(-5.7 \times 10^4 \text{ kg} \cdot \text{m/s})}{3.8 \text{ s}}$$

$$= -1.5 \times 10^4 \text{ N}$$
Average Force (Cont.)

Substitute $\Delta t_{\text{hitting a wall}} = 0.22\ \text{s}$

$$F_{\text{hitting a wall}} = \frac{(-5.7 \times 10^4\ \text{kg} \cdot \text{m/s})}{0.22\ \text{s}}$$

$$= -2.6 \times 10^5\ \text{N}$$
Average Force (Cont.)

Step 3: Evaluate the Answer
Average Force (Cont.)

Are the units correct?
Force is measured in newtons.

Does the direction make sense?
Force is exerted in the direction opposite to the velocity of the car and thus, is negative.
Is the magnitude realistic?

People weigh hundreds of newtons, so it is reasonable that the force needed to stop a car would be in thousands of newtons. The impulse is the same for all three stops. Thus, as the stopping time is shortened by more than a factor of 10, the force is increased by more than a factor of 10.
The steps covered were:

**Step 1: Analyze the Problem**

Sketch the system.

Include a coordinate axis and select the positive direction to be the direction of the velocity of the car.

Draw a vector diagram for momentum and impulse.
The steps covered were:

**Step 2: Solve for the Unknown**

Determine the initial momentum, \( p_i \), before the crash.

Determine the final momentum, \( p_f \), after the crash.

Apply the impulse-momentum theorem to obtain the force needed to stop the vehicle.

**Step 3: Evaluate the Answer**
Angular Momentum

- The angular velocity of a rotating object changes only if torque is applied to it.
- This is a statement of Newton’s law for rotating motion, $\tau = I \Delta \omega / \Delta t$. 
• This equation can be rearranged in the same way as Newton’s second law of motion was, to produce \( \tau \Delta t = I \Delta \omega \).

• The left side of this equation is the angular impulse of the rotating object and the right side can be rewritten as \( \Delta \omega = \omega_f - \omega_i \).
Angular Momentum (Cont.)

- The **angular momentum** of an object is equal to the product of a rotating object’s moment of inertia and angular velocity.

**Angular Momentum** \( L = I\omega \)

- Angular momentum is measured in kg·m²/s.
Angular Momentum (Cont.)

- Just as the linear momentum of an object changes when an impulse acts on it, the angular momentum of an object changes when an angular impulse acts on it.

- Thus, the angular impulse on the object is equal to the change in the object’s angular momentum, which is called the angular impulse-angular momentum theorem.
Angular Momentum (Cont.)

• The angular impulse-angular momentum theorem is represented by the following equation.

Angular Impulse-Angular Momentum Theorem

\[ \tau \Delta t = L_f - L_i \]
Angular Momentum (Cont.)

• If there are no forces acting on an object, its linear momentum is constant.

• If there are no torques acting on an object, its angular momentum is also constant.

• Because an object’s mass cannot be changed, if its momentum is constant, then its velocity is also constant.
In the case of angular momentum, however, the object’s angular velocity does not remain constant. This is because the moment of inertia depends on the object’s mass and the way it is distributed about the axis of rotation or revolution.
Angular Momentum (Cont.)

- Thus, the angular velocity of an object can change even if no torques are acting on it.
- Observe the animation.
Angular Momentum (Cont.)

• How does she start rotating her body?

• She uses the diving board to apply an external torque to her body.

• Then, she moves her center of mass in front of her feet and uses the board to give a final upward push to her feet.

• This torque acts over time, $\Delta t$, and thus increases the angular momentum of the diver.
Angular Momentum (Cont.)

- Before the diver reaches the water, she can change her angular velocity by changing her moment of inertia. She may go into a tuck position, grabbing her knees with her hands.
Angular Momentum (Cont.)

By moving her mass closer to the axis of rotation, the diver decreases her moment of inertia and increases her angular velocity.
When she nears the water, she stretches her body straight, thereby increasing the moment of inertia and reducing the angular velocity.

As a result, she goes straight into the water.
A. Momentum is the ratio of change in velocity of an object to the time over which the change happens.

B. Momentum is the product of the average force on an object and the time interval over which it acts.

C. Momentum of an object is equal to the mass of the object times the object’s velocity.

D. Momentum of an object is equal to the mass of the object times the change in the object’s velocity.
Reason: The momentum of an object is equal to the mass of the object times the object’s velocity $p = mv$.

Momentum is measured in kg·m/s.
Mark and Steve are playing baseball. Mark hits the ball with an average force of 6000 N and the ball snaps away from the bat in 0.2 m/s. Steve hits the same ball with an average force of 3000 N and the ball snaps away in 0.4 m/s. Which of the following statements is true about the impulse given to the ball in both the shots is true?
A. The impulse given to the ball by Mark is twice the impulse given by Steve.

B. The impulse given to the ball by Mark is four times the impulse given by Steve.

C. The impulse given to the ball by Mark is the same as the impulse given by Steve.

D. The impulse given to the ball by Mark is half the impulse given by Steve.
Answer

Reason: Impulse is the product of the average force on an object and the time interval over which it acts. Since the product of the average force on the ball and the time interval of the impact in both the shots is the same, the impulse given to the ball by Mark is the same as the impulse given by Steve.
Answer

Reason: Impulse given to the ball by Mark
= (6000 N) (0.2×10^{-3} s)
= 1.2 N·s

Impulse given to the ball by Steve
= (3000 N) (0.4×10^{-3} s)
= 1.2 N·s
In a baseball game, a pitcher throws a ball with a mass of 0.145 kg with a velocity of 40.0 m/s. The batter hits the ball with an impulse of 14.0 kg·m/s. Given that the positive direction is toward the pitcher, what is the final momentum of the ball?

A. \( p_f = (0.145 \text{ kg})(40.0 \text{ m/s}) + 14.0 \text{ kg} \cdot \text{m/s} \)

B. \( p_f = (0.145 \text{ kg})(-40.0 \text{ m/s}) - 14.0 \text{ kg} \cdot \text{m/s} \)

C. \( p_f = (0.145 \text{ kg}) (40.0 \text{ m/s}) - 14.0 \text{ kg} \cdot \text{m/s} \)

D. \( p_f = (0.145 \text{ kg})(-40.0 \text{ m/s}) + 14.0 \text{ kg} \cdot \text{m/s} \)
Reason: By the impulse-momentum theorem,
\[ p_f = p_i + F \Delta t \]
where, \( p_i = m v_i \)
\( F \Delta t = \text{impulse} \)
\[ p_f = m v_i + \text{impulse} \]
Answer

Reason: Since the positive direction is toward the pitcher, $v_i$ is taken as negative as the ball is moving away from the pitcher before the batter hits the ball. The impulse is positive because the direction of the force is toward the pitcher.

Therefore, $p_f = mv_i + \text{impulse} = (0.145 \text{ kg})(-40 \text{ m/s}) + 14 \text{ kg} \cdot \text{m/s}$. 
Define the angular momentum of an object.

A. The angular momentum of an object is the ratio of change in the angular velocity of the object to the time over which the change happens.

B. The angular momentum of an object is equal to the mass of the object times the object’s angular velocity.

C. The angular momentum of an object is equal to the moment of inertia of the object times the object’s angular velocity.

D. The angular momentum of an object is equal to the moment of inertia of the object times the change in the object’s angular velocity.
Answer

**Reason:** The angular momentum of an object is equal to the product of the object’s moment of inertia and the object’s angular velocity.

\[ L = I\omega \]

The angular momentum is measured in kg·m\(^2\)/s.
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MAIN IDEA

In a closed, isolated system, linear momentum and angular momentum are conserved.

Essential Questions

• How does Newton’s third law relate to conservation of momentum?
• Under which conditions is momentum conserved?
• How can the law of conservation of momentum and the law of conservation of angular momentum help explain the motion of objects?
Review Vocabulary

- **momentum** the product of an object’s mass and the object’s velocity

New Vocabulary

- Closed system
- Isolated system
- Law of conservation of momentum
- Law of conservation of angular momentum
Two-Particle Collisions

Click image to view the movie.
Under what conditions is the momentum of the system of two balls conserved?

The first and most obvious condition is that no balls are lost and no balls are gained. Such a system, which does not gain or lose mass, is said to be a closed system.
Momentum in a Closed, Isolated System
(cont.)

- The second condition is that the forces involved are internal forces; that is, there are no forces acting on the system by objects outside of it.

- When the net external force on a closed system is zero, the system is described as an isolated system.
Momentum in a Closed, Isolated System (cont.)

- No system on Earth can be said to be absolutely isolated, because there will always be some interactions between a system and its surroundings.

- Often, these interactions are small enough to be ignored when solving physics problems.
• Systems can contain any number of objects, and the objects can stick together or come apart in a collision.

• Under these conditions, the **law of conservation of momentum** states that the momentum of any closed, isolated system does not change.

• This law will enable you to make a connection between conditions, before and after an interaction, without knowing any of the details of the interaction.
Speed

A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two cars move together immediately after the collision?
Step 1: Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate system.
- Sketch the situation showing the “before” and “after” states.
- Draw a vector diagram for the momentum.
Conservation of Momentum

**Speed (cont.)**

Identify the known and unknown variables.

**Known:**

- \( m_C = 1875 \text{ kg} \)
- \( v_{Ci} = +23 \text{ m/s} \)
- \( m_D = 1025 \text{ kg} \)
- \( v_{Di} = +17 \text{ m/s} \)

**Unknown:**

- \( v_f = ? \)
Step 2: Solve for the Unknown
Momentum is conserved because the ice makes the total external force on the cars nearly zero.

\[
p_i = p_f
\]

\[
p_{Ci} + p_{Di} = p_{Cf} + p_{Df}
\]

\[
m_C v_{Ci} + m_D v_{Di} = m_C v_{Cf} + m_D v_{Df}
\]
Because the two cars stick together, their velocities after the collision, denoted as $v_f$, are equal.

$$v_{Cf} = v_{Df} = v_f$$

$$m_C v_{Ci} + m_D v_{Di} = (m_C + m_D) v_f$$
Solve for $v_f$.

$$v_f = \frac{(m_Cv_{Ci} + m_Dv_{Di})}{(m_C + m_D)}$$
Substitute $m_C = 1875 \text{ kg}$, $v_{Ci} = +23 \text{ m/s}$, $m_D = 1025 \text{ kg}$, $v_{Di} = +17 \text{ m/s}$

$$v_f = \frac{(1875 \text{ kg})(+23 \text{ m/s}) + (1025 \text{ kg})(+17 \text{ m/s})}{(1875 \text{ kg} + 1025 \text{ kg})}$$

$$= + 21 \text{ m/s}$$
Step 3: Evaluate the Answer
Are the units correct?

Velocity is measured in m/s.

Does the direction make sense?

\( v_i \) and \( v_f \) are both in the positive direction; therefore, \( v_f \) should be positive.
Is the magnitude realistic?

The magnitude of $v_f$ is between the initial speeds of the two cars, but closer to the speed of the more massive one, so it is reasonable.
The steps covered were:

**Step 1: Analyze the Problem**

Define the system.

Establish a coordinate system.

Sketch the situation showing the “before” and “after” states.

Draw a vector diagram for the momentum.
The steps covered were:

**Step 2:** Solve for the Unknown

**Step 3:** Evaluate the Answer
Recoil

• The momentum of a baseball changes when the external force of a bat is exerted on it. The baseball, therefore, is not an isolated system.

• On the other hand, the total momentum of two colliding balls within an isolated system does not change because all forces are between the objects within the system.
Recoil (cont.)

- Observe the animation below.
Assume that a girl and a boy are skating on a smooth surface with no external forces. They both start at rest, one behind the other. Skater C, the boy, gives skater D, the girl, a push. Find the final velocities of the two in-line skaters.
Recoil (cont.)

- After clashing with each other, both skaters are moving, making this situation similar to that of an explosion. Because the push was an internal force, you can use the law of conservation of momentum to find the skaters’ relative velocities.

- The total momentum of the system was zero before the push. Therefore, it must be zero after the push.
Recoil (cont.)

Before

\[ \mathbf{p}_{\text{Ci}} + \mathbf{p}_{\text{Di}} = \mathbf{0} \]

\[ \mathbf{p}_{\text{Cf}} = \mathbf{m}_C \mathbf{v}_{\text{Cf}} \]

After

\[ \mathbf{p}_{\text{Cf}} + \mathbf{p}_{\text{Df}} \]

\[ \mathbf{p}_{\text{Cf}} = -\mathbf{p}_{\text{Df}} \]

\[ \mathbf{m}_C \mathbf{v}_{\text{Cf}} = -\mathbf{m}_D \mathbf{v}_{\text{Df}} \]
The coordinate system was chosen so that the positive direction is to the left.

The skaters’ momentums after the push are equal in magnitude but opposite in direction. The backward motion of skater C is an example of recoil.
Recoil (cont.)

- Are the skaters’ velocities equal and opposite?
- The last equation, for the velocity of skater C, can be rewritten as follows:

\[ \mathbf{v}_{Cf} = \left( \frac{-m_D}{m_C} \right) \mathbf{v}_{Df} \]
Recoil (cont.)

- The velocities depend on the skaters’ relative masses. The less massive skater moves at the greater velocity.

- Without more information about how hard skater C pushed skater D, you cannot find the velocity of each skater.
How does a rocket in space change its velocity?

The rocket carries both fuel and oxidizer. When the fuel and oxidizer combine in the rocket motor, the resulting hot gases leave the exhaust nozzle at high speed.
If the rocket and chemicals are the system, then the system is a closed system.

The forces that expel the gases are internal forces, so the system is also an isolated system.

Thus, objects in space can accelerate using the law of conservation of momentum and Newton’s third law of motion.
Propulsion in Space (Cont.)

- A NASA space probe, called Deep Space 1, performed a flyby of asteroid Braille in 1999.
- The most unusual of the 11 new technologies on board was an ion engine that exerts as much force as a sheet of paper resting on a person’s hand.
In a traditional rocket engine, the products of the chemical reaction taking place in the combustion chamber are released at high speed from the rear.

In the ion engine, however, xenon atoms are expelled at a speed of 30 km/s, producing a force of only 0.092 N.
• How can such a small force create a significant change in the momentum of the probe?

• Instead of operating for only a few minutes, as the traditional chemical rockets do, the ion engine can run continuously for days, weeks, or months. Therefore, the impulse delivered by the engine is large enough to increase the momentum.
Two-Dimensional Collisions

- Until now, you have looked at momentum in only one dimension.

- The law of conservation of momentum holds for all closed systems with no external forces.

- It is valid regardless of the directions of the particles before or after they interact.

- But what happens in two or three dimensions?
Two-Dimensional Collisions (cont.)

- Consider the two billiard balls to be the system.
- The original momentum of the moving ball is $p_{Ci}$ and the momentum of the stationary ball is zero.
- Therefore, the momentum of the system before the collision is equal to $p_{Ci}$. 
Two-Dimensional Collisions (cont.)

- After the collision, both billiard balls are moving and have momenta.
- As long as the friction with the tabletop can be ignored, the system is closed and isolated.
- Thus, the law of conservation of momentum can be used. The initial momentum equals the vector sum of the final momenta. So:

\[ p_{Ci} = p_{Cf} + p_{Df} \]
Two-Dimensional Collisions (cont.)

- The equality of the momenta before and after the collision also means that the sum of the components of the vectors before and after the collision must be equal.

- Suppose the \( x \)-axis is defined to be in the direction of the initial momentum, then the \( y \)-component of the initial momentum is equal to zero.

- Therefore, the sum of the final \( y \)-components also must be zero.

\[
p_{Cf, y} + p_{Df, y} = 0
\]
Two-Dimensional Collisions (cont.)

- The $y$-components are equal in magnitude but are in the opposite direction and, thus, have opposite signs. The sum of the horizontal components is equal to the initial momentum.

$$p_{Ci} = p_{Cf, x} + p_{Df, x}$$
Conservation of Angular Momentum

• Like linear momentum, angular momentum can be conserved.

• The law of conservation of angular momentum states that if no net external torque acts on an object, then its angular momentum does not change.

• This is represented by the following equation.

\[ L_f = L_i \]

• An object’s initial angular momentum is equal to its final angular momentum.
Earth spins on its axis with no external torques. Its angular momentum is constant.

Thus, Earth’s angular momentum is conserved.

As a result, the length of a day does not change.
Conservation of Angular Momentum (cont.)

• When an ice skater pulls in his arms, he begins spinning faster.
• Without an external torque, his angular momentum does not change; that is, $L = I\omega$ is constant.
Thus, the ice-skater’s increased angular velocity must be accompanied by a decreased moment of inertia.

By pulling his arms close to his body, the ice-skater brings more mass closer to the axis of rotation, thereby decreasing the radius of rotation and decreasing his moment of inertia.

\[ L_i = L_f \]

thus, \[ I_i \omega_i = I_f \omega_f \]

\[ \frac{\omega_f}{\omega_i} = \frac{I_f}{I_i} \]
Because frequency is \( f = \omega / 2\pi \), the above equation can be rewritten as follows:

\[
\frac{2\pi (f_f)}{2\pi (f_i)} = \frac{l_f}{l_i}
\]

thus, \( \frac{f_f}{f_i} = \frac{l_f}{l_i} \)

Notice that because \( f, \omega, \) and \( l \) appear as ratios in these equations, any units may be used, as long as the same unit is used for both values of the quantity.
• If a torque-free object starts with no angular momentum, it must continue to have no angular momentum.

• Thus, if part of an object rotates in one direction, another part must rotate in the opposite direction.

• For example, if you switch on a loosely held electric drill, the drill body will rotate in the direction opposite to the rotation of the motor and bit.
Because of the conservation of angular momentum, the direction of rotation of a spinning object can be changed only by applying a torque.

If you played with a top as a child, you may have spun it by pulling the string wrapped around its axle.
Tops and Gyroscopes (cont.)

- When a top is vertical, there is no torque on it, and the direction of its rotation does not change.
If the top is tipped, as shown in the figure, a torque tries to rotate it downward. Rather than tipping over, however, the upper end of the top revolves, or precesses slowly about the vertical axis.
A gyroscope, such as the one shown in the figure, is a wheel or disk that spins rapidly around one axis while being free to rotate around one or two other axes.
The direction of its large angular momentum can be changed only by applying an appropriate torque. Without such a torque, the direction of the axis of rotation does not change.
Gyroscopes are used in airplanes, submarines, and spacecraft to keep an unchanging reference direction.

Giant gyroscopes are used in cruise ships to reduce their motion in rough water. Gyroscopic compasses, unlike magnetic compasses, maintain direction even when they are not on a level surface.
During a badminton match, as a player hits the shuttlecock, the head of the shuttlecock separates from the feathers and falls down. Is the momentum conserved?
No, momentum of a system is conserved only if the following conditions are satisfied.

(i) No mass is lost or gained.

(ii) There are no external forces acting on the system.
The momentum of the system is conserved if both the above conditions are satisfied. If only one condition is satisfied, momentum is not conserved. In this case, both the conditions are not satisfied. When the player hits the shuttlecock, an external force is applied. Hence, the second condition is not satisfied. Further, since the head of the shuttlecock separates from the feathers and falls down, mass is lost and hence the first condition is not satisfied. So, momentum is not conserved.
A goalkeeper kicks a ball approaching the goal post. Is the momentum of the ball conserved?

A. No, because the system is not closed.
B. No, because the system is not isolated.
C. Yes, because the total momentum of the ball before the kick is equal to the total momentum of the ball after the kick.
D. Yes, because the impulse experienced by the ball is zero.
Reason: For momentum to be conserved, the system should be closed and isolated. That is, no mass is lost or gained and there are no forces acting on the system by the object outside of it. However, as the goalkeeper kicks the ball, he applies an external force to the ball and hence the system does not remain isolated. So, momentum is not conserved.
In a billiards game, the cue ball hits a stationary red ball. $p_{Ci}$ and $p_{Ri}$ are the initial momentums of the cue ball and red ball respectively, and $p_{Cf}$ and $p_{Rf}$ are the final momentums of the cue ball and red ball respectively. If the y-axis is defined to be in the direction of the momentum of the cue ball and the friction of the tabletop is ignored, which of the following vector sums is correct?
A. $p_{Ci} = p_{Cf, x} + p_{Rf, x}$
B. $p_{Ci} = p_{Cf, x} + p_{Rf, y}$
C. $p_{Ci} = p_{Cf, y} + p_{Rf, y}$
D. $p_{Ci} = p_{Cf, y} + p_{Rf, x}$
Reason: By the law of conservation of momentum,

\[ \mathbf{p}_{Ci} + \mathbf{p}_{Ri} = \mathbf{p}_{Cf} + \mathbf{p}_{Rf} \]

Since the red ball is stationary, \( \mathbf{p}_{Ri} = 0 \).

Now, since the y-axis is defined to be in the direction of the initial momentum of the cue ball, the x-component of the initial momentum is zero. Therefore, the sum of the final x-components would also be zero. That is, \( \mathbf{p}_{Cf, x} + \mathbf{p}_{Rf, x} = 0 \).

\[ \therefore \mathbf{p}_{Ci} = \mathbf{p}_{Cf, y} + \mathbf{p}_{Rf, y} \]
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Resources

Physics Online
Study Guide
Chapter Assessment Questions
Standardized Test Practice
The impulse on an object is the average net force exerted on the object multiplied by the time interval over which the force acts.

\[ \text{impulse} = F \Delta t \]

The momentum of an object is the product of its mass and velocity and is a vector quantity.

\[ p = mv \]
When solving a momentum problem, first define the objects in the system and examine their momentum before and after the event. The impulse on an object is equal to the change in momentum of the object.

\[ F \Delta t = p_f - p_i \]
The angular momentum of a rotating object is the product of its moment of inertia and its angular velocity.

\[ L = I \omega \]

The angular impulse-angular momentum theorem states that the angular impulse on an object is equal to the change in the object’s angular momentum.

\[ \tau \Delta t = L_f - L_i \]
According to Newton’s third law of motion and the law of conservation of momentum, the forces exerted by colliding objects on each other are equal in magnitude and opposite in direction.

Momentum is conserved in a closed, isolated system.

\[ p_f = p_i \]
The law of conservation of momentum relates the momentum of objects before and after a collision. Use vector analysis to solve momentum-conservation problems in two dimensions. The law of conservation of angular momentum states that if there are no external torques acting on a system, then the angular momentum is conserved.

\[ L_i = L_f \]
Because angular momentum is conserved, the direction of rotation of a spinning object can be changed only by applying a torque.
A 1500-kg car hits a wall with a velocity of 20 m/s and immediately comes to rest. What is the final momentum of the car?

A. (1500 kg)(20 m/s)
B. (1500 kg)(–20 m/s)
C. 0 kg·m/s
D. (1500 kg)(20 m/s)^2
Reason: Momentum of an object is equal to the mass of the object times the object’s velocity.

\[ p_f = m v_f \]

Since the car comes to rest, \( v_f = 0 \).

Therefore,

\[ p_f = (1500 \text{ kg})(0 \text{ m/s}) = 0 \text{ kg}\cdot\text{m/s}. \]
If no net force is acting on an object, its velocity remains the same. Explain how the angular velocity can change when no torque is acting on an object.
**Answer:** If there is no torque acting on an object, its angular momentum is constant. In case of angular momentum, the object’s angular velocity may not remain the same. An object’s angular momentum depends on both its angular velocity and its moment of inertia. Therefore, changing the moment of inertia of an object also changes the angular velocity of that object, even when there is no net torque on the object.
Why does the angular velocity of a planet increase when that planet is closer to the Sun? Explain on the basis of angular momentum.
Answer: The torque on a planet’s revolution around the Sun is zero, since there is no component of force parallel to the orbital motion. Therefore, the planet’s angular momentum remains constant throughout its orbital motion. When the distance between the planet and the Sun decreases, the planet’s moment of inertia of revolution in the orbit about the Sun also decreases. In order for the planet’s angular momentum to be conserved, the angular velocity of the planet must increase.
Can the momenta of a bicycle and a car be the same?

**Answer:** Yes, because momentum of an object is equal to the mass of the object times the object’s velocity. A car does have much greater mass than a bicycle. However, if the bicycle has a much higher velocity than the car, then the two could have the same momenta.
How can an ice skater increase his angular velocity without external torque?

A. by decreasing his moment of inertia
B. by increasing the angular momentum
C. by increasing the linear velocity
D. by increasing his moment of inertia
Reason: Without an external torque, the angular momentum does not change. Thus, the ice skater’s increased angular velocity must be accompanied by a decreased moment of inertia. By pulling his arms close to his body, the ice skater brings more mass closer to the axis of rotation, thereby decreasing his moment of inertia.
When a star that is much larger than the Sun nears the end of its lifetime, it begins to collapse, but continues to rotate. Which of the following describes the conditions of the collapsing star’s moment of inertia (I), angular momentum (L), and angular velocity (ω)?

A.  I increases, L stays constant, ω decreases

B.  I decreases, L stays constant, ω increases

C.  I increases, L increases, ω increases

D.  I increases, L increases, ω constant
A 40.0-kg ice skater glides with a speed of 2.0 m/s toward a 10.0-kg sled at rest on the ice. The ice skater reaches the sled and holds on to it. The ice skater and the sled then continue sliding in the same direction in which the ice skater was originally skating. What is the speed of the ice skater and the sled after they collide?

A. 0.4 m/s  
B. 0.8 m/s  
C. 1.6 m/s  
D. 3.2 m/s
A bicyclist applies the brakes and slows the motion of the wheels. The angular momentum of each wheel then decreases from 7.0 kg·m²/s to 3.5 kg·m²/s over a period of 5.0 s. What is the angular impulse on each wheel?

A. -0.7 kg·m²/s  
B. -1.4 kg·m²/s  
C. -2.1 kg·m²/s  
D. -3.5 kg·m²/s
A 45.0-kg ice skater stands at rest on the ice. A friend tosses the skater a 5.0-kg ball. The skater and the ball then move backward across the ice with a speed of 0.50 m/s. What was the speed of the ball at the moment just before the skater caught it?

A. 2.5 m/s
B. 3.0 m/s
C. 4.0 m/s
D. 5.0 m/s
What is the difference in momentum between a 50.0-kg runner moving at a speed of 3.00 m/s and a 3.00 \times 10^3-kg truck moving at a speed of only 1.00 m/s?

A. 1275 kg·m/s
B. 2550 kg·m/s
C. 2850 kg·m/s
D. 2950 kg·m/s
If It Looks Too Good To Be True

Beware of answer choices in multiple-choice questions that seem ready-made and obvious. Remember that only one answer choice for each question is correct. The rest are made up by test-makers to distract you. This means that they might look very appealing. Check each answer before making your final selection.
Force on a Baseball
Average Force

Vector diagram

\[ p_i \quad \text{Impulse} \quad p_f \]

2200 kg

94 km/h
Diver Changing Her Moment of Inertia
Two-Particle Collisions
Chapter 9

Momentum and Its Conservation

Speed

Before (initial)

C

D

$v_{Ci}$

$v_{Di}$

After (final)

C

D

$v_{Cf} = v_{Df} = v_f$

Vector diagram

$p_{Ci}$

$p_{Di}$

$p_f$

$p_i = p_{Ci} + p_{Di}$
An Astronaut Firing a Thruster Pistol

Before (initial)

\[ v_i = 0.0 \text{ m/s} \]

\[ \mathbf{p}_i \]

After (final)

\[ v_D \]

\[ D \]

\[ v_C \]

\[ \mathbf{p}_f = \mathbf{p}_{Cf} + \mathbf{p}_{Df} \]

Vector Diagram

\[ \mathbf{p}_{Df} \]

\[ \mathbf{p}_{Cf} \]
Momentum and Its Conservation

Billiard Balls Colliding
A Top

Rotational axis

Vertical

Spin

Center of mass

Precession due to torque

Gravitational force

Angle of rotational axis with the vertical

Pivot point
Gyroscope
A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21 s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes, or in 0.22 s if it hits a concrete wall. What average force is exerted on the vehicle in each of these stops?
A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two cars move together immediately after the collision?